Pyramids as Inclined Planes

Henry Petroski

The last Egyptian hieroglyph is said to have been inscribed late in the 4th century A.D., but serious study of Egyptian culture by Westerners did not begin until the 17th century. It was then that the first relatively precise measurements of the Great Pyramid were made. Near the close of the 18th century, the young French republic sent to Egypt—under the command of Napoleon—a large expedition that included a "scientific and artistic commission." The expedition not only resulted in volumes of scholarship but also led to the accidental uncovering of the Rosetta Stone, which promised to be a key to deciphering hieroglyphs. Thus the foundations of Egyptology were laid, and for the past two centuries scholars and amateurs alike have built it into an edifice. Among the many intriguing open questions about the ancient culture has long been, how were the pyramids built? There have been many answers, most of which raise more questions.

A recent issue of *Technology and Culture*, the international quarterly of the Society for the History of Technology, carried a remarkable research note on "probable construction methods employed" in building the Great Pyramid at Giza. According to James Frederick Edwards, a chartered consultant engineer from Manchester, England, the ancient Egyptians probably did not construct special ramps or use incremental levering techniques to raise large blocks of stone to their final resting places. Rather, he proposes a "more logical and practical alternative methodology," in which the sides of the incomplete pyramid itself were used as inclined planes up which the individual blocks were hauled on sledges.

Using little more than empirical evidence and elementary engineering calculations, Edwards demonstrates that such a hauling system was not only physically possible but also more probable than previously proposed methods. He concludes that, using the system he describes, the Egyptians could have completed the entire structure of the Great Pyramid within the 23-year reign of King Khufu with a force of no greater than about 10,000 people working on the construction site during peak activity. In fact, Edwards's analysis is so convincing that it has led this reader to imagine that an even more efficient process could have been followed, one that was less onerous and more worker friendly than those usually depicted.

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Heave Ho to Ramps and Levers

Edwards begins his analysis by discrediting the ramp and lever theories. According to him, "the principal theory is that a massive ramp was built against one full face of the pyramid, and was lengthened as construction proceeded." With a grade of 1 in 10, "considered the most practical" according to Edwards, such a ramp would have reached 1.5 kilometers in length and contained more than three times the material in the pyramid itself. Edwards's skepticism should resonate with anyone who has seen the ramp employed at the World Trade Center site during the removal of debris from atop the bedrock at ground zero. The depth of the hole was only about one-eighth the height of the Great Pyramid. Were the hole as deep as the pyramid is high, it would clearly have been impossible to reach its bottom via a single straight ramp. It is for the same reason that deep open-pit mines are ringed with spiral ramps.

Edwards is incredulous that a spiral ramp was employed at Giza, however, since the relative narrowness of such a ramp would have presented difficulties both for two-way traffic (teams dragging stones on sledges up and others taking empty sledges down) and when negotiating the turns at each corner of the pyramid. Similarly, he finds fault with any scheme using levers, arguing that it would be slow and tricky, to say the least. He believes both ramps and levers "would have been inefficient in their deployment of personnel, for in both cases the haulers and lifters would have had to ascend and descend the pyramid structure as part of each elevating cycle." He estimates that, when the pyramid was half finished, "the elevating cycle for one core block would have been 40 minutes using a straight ramp and seven hours using levers." And he proposes methods that he believes to be easier and quicker.

Figure 1. Over time, various theories have been developed to explain the means by which Egyptians lifted stone blocks weighing approximately a metric ton each into position to form pyramids. According to engineer James Frederick Edwards, complex solutions such as ramps or levers are unnecessary. The pyramids themselves form inclined planes with steep angles, which when faced (as shown near the top of Khafre's pyramid, *rear*) could have been used to hoist the blocks.

The fundamental question Edwards poses is, "why build separate ramps when the pyramid has four inclined planes as an integral part of its structure?" To show that 52-degree slopes (faced as they rise to provide as regular a surface as possible) are not too steep for gangs of workers to drag stones up, he deduces some physical parameters from ancient and contemporary evidence and performs an elementary calculation (using mathematics no more complicated than trigonometry) relating to the forces involved. Among the critical parameters is the coefficient of friction between a wooden sledge bearing a building block and the stone face of the incomplete pyramid.

Friction always opposes motion, so workers hauling stones up inclines must overcome not only the proportion of the weight acting down the slope but also the friction force between the sledge and the incline. The ratio of the friction force itself to the force bearing down squarely on the surface over which the load is being dragged is known as the coefficient of friction. Edwards appeals to "recent experiments" at Karnak Temple, in which "it was found that three men could pull a sledge-mounted block weighing one tonne [1,000 kilograms] over a stone surface that had been lubricated with water to reduce the effects of friction." To estimate the coefficient of friction, Edwards makes an assumption about how much force an adult male can exert on a hauling rope. He takes this to be 68 kilograms, or 90 percent of average body weight. A simple calculation then gives the coefficient to be $3 \times 68/1{,}000$, or about 0.2.

To check the reasonableness of his result, Edwards looks to an ancient wall painting in the Twelfth Dynasty tomb at Deir el-Bersha, which depicts the hauling on a sledge of a massive statue of the Egyptian nobleman and tomb-occupant Djehutihotep. Using the coefficient of friction derived from the experiment at Karnak and the known weight (58 tonnes) of the Djehutihotep statue, Edwards concludes that it would have taken some 174 men to pull the load. Since the wall painting shows a team of 172 men pulling the statue, Edwards concludes that his assumptions and results are reasonable. He then proceeds to calculate how many men it would take to haul a building block on a sledge up the side of the pyramid at Giza.

Calculating how much force it takes to pull an object up an inclined plane is elementary, involving only the weight of the load, the coefficient of friction and the angle of the incline. For the purposes of his calculation, Edwards assumes the weight of the block being hauled to be 2 tonnes, which he takes to be representative of the core blocks in the pyramid, and uses the same coefficient of friction as at Karnak. He also recognizes that the workers would have to pull also against the weight of the sledge and that of the (8-centimeter diameter) rope, which he estimates at 0.3 tonnes and 0.5 tonnes, respectively. (Edwards documents such numerical details in footnotes.) Given these assumptions and the coefficient of friction established earlier, Edwards deduces that it would take a team of 50 men to drag a single block up the side of the pyramid, perhaps supplemented by a few pushers to start the load moving. (The nature of friction is that the force required to keep something moving is less than that needed to start it moving.)

Edwards assumes that the pyramid was constructed tier by tier, so that it rose by the height of one core block (typically about a meter and a half) at a time. Thus, each newly completed level would have provided a fresh flat surface—a plateau—to which stones could be hauled and pushed into position to raise another level. As the pyramid rose, the outer casing blocks would have been put into position on the faces up which the hauling occurred. These casing blocks "would have been dressed by the stonemasons on their angled outside surfaces in order to provide a reasonably smooth surface for the blocks to be hauled up on." Oversized outer blocks would have been used, so that once all the stones were in place the scars of construction could be erased by a final dressing.

Edwards imagines each stone being hauled up the incline by a 50-man team working on the plateau formed on the partially completed pyramid. By remaining atop the pyramid throughout the workday ("where they may indeed have lived during the more intensive periods of construction"), the workers did not have to waste time returning empty-handed to the base of the pyramid after each block was raised. He further posits that a number of teams would have been working on the plateau simultaneously, each being assigned to a "slipway" 5 meters wide so as not to interfere with other teams working in adjacent slipways. When the pyramid had reached a quarter of its height, the 37-meter high plateau would have been about 173 meters on a side, thus allowing for 35 slipways. Given the width of the plateau compared to the length of the incline (47 meters), two teams could have worked without interference, simultaneously hauling blocks up two opposite sides of the pyramid and placing them on the plateau from the center out. As the pyramid rose, the working space would have diminished, of course, and so would have the number of teams that could simultaneously work atop it. Nevertheless, at the half height of the pyramid, Edwards estimates that by his scheme it would have taken less than 3 minutes to haul a block from ground to plateau.

Of course, getting the blocks up the incline was only one aspect of the construction process. Once a block had reached the plateau, it had to be moved over to its proper place, unloaded from the sledge, the empty sledge lowered back to the ground, the rope undone and attached to a waiting sledge with another block on it, and the cycle repeated. Edwards estimates that on average it would have taken one hour to execute this cycle for each block. With multiple teams working atop plateaus, the volume of the pyramid, at least in its lower stages, could have grown faster than a block a minute. Even allowing for the complications of heavier blocks and the more complicated geometry associated with burial chambers and passages, Edwards believes that the entire pyramid (with its estimated 2.3 million individual blocks of stone) could have been completed in 23 years, and with a workforce—including those necessary to quarry the core stone and move it to the construction site, but not including those transporting outer casing and other special stone from greater distances—at no time exceeding about 10,000 people.

Nevertheless, Edwards admits that as the pyramid approached its apex his scheme would have been increasingly difficult to implement. Reduced plateau size would have required hauling teams to work in slipways that were shorter than the incline up which the blocks had to be pulled. More ropes could have been used, but only up to a point. "Technically," Edwards concedes, "the final 10 percent (by volume) of the pyramid would have been the most difficult to construct."

Variations on a Theme

Although Edwards's speculation about the great construction project does away with the need for ramps and levers to raise the blocks to their final resting height, it does not say much about the endurance or commitment of the workers hauling stones up the incline and moving them into position on the plateau. According to his scenario, the team would be engaged in a very repetitive process. Presumably there was some relief, since the entire team of 50 would have been needed only during the actual hauling process. There could have been periods of rest for some of them while subteams moved and positioned the block on the plateau, and others lowered the empty sledge to begin the cycle anew. But mostly it was just hard work.

It is possible to imagine a variation on Edwards's scheme, one in which the total amount of hauling work would be significantly decreased. In fact, some of the workers could engage in productive work in raising the blocks of stone while partaking in what might almost be considered recreation. This scheme would make use of the fact that with the raising of each stone comes a sledge, which must return to the base of the pyramid to begin a new cycle. In Edwards's approach, lowering the sledge would have required effort, for the rope would have had to be let out in a controlled fashion, lest the sledge accelerate out of control down the incline. A runaway sledge could have crashed into blocks on other sledges on their way up, could have injured workers on the ground and could have destroyed itself. Thus, care would have been required.

The process of lowering an empty sledge properly would have revealed some interesting things. For one, it would have taken about three men pulling on the rope to hold the sledge under control when it began its descent. In addition, as the sledge moved down the incline, it would have become increasingly difficult for the workers to keep it from accelerating. In fact, when the pyramid approached about half its final height, it might have taken an additional half dozen men holding onto the rope to maintain control. This is because as more rope was played out, the workers would have had to hold back not only the weight of the sledge but also that of the increasing length (and hence weight) of rope attached to it. There could have been an alternative.

If in fact two sledges were connected with a length of rope that reached from the base on one side of the truncated pyramid to the edge of the plateau on the other side, then a counterweight system could have been employed to advantage. To raise a block of stone, a number of workers equal to, say, half the weight of the stone (about a dozen men) could sit on the empty sledge, as if ready to take a toboggan ride down the other side of the pyramid. Because a greater weight plus the weight of the rope had to be raised, and because it takes an extra effort to overcome the

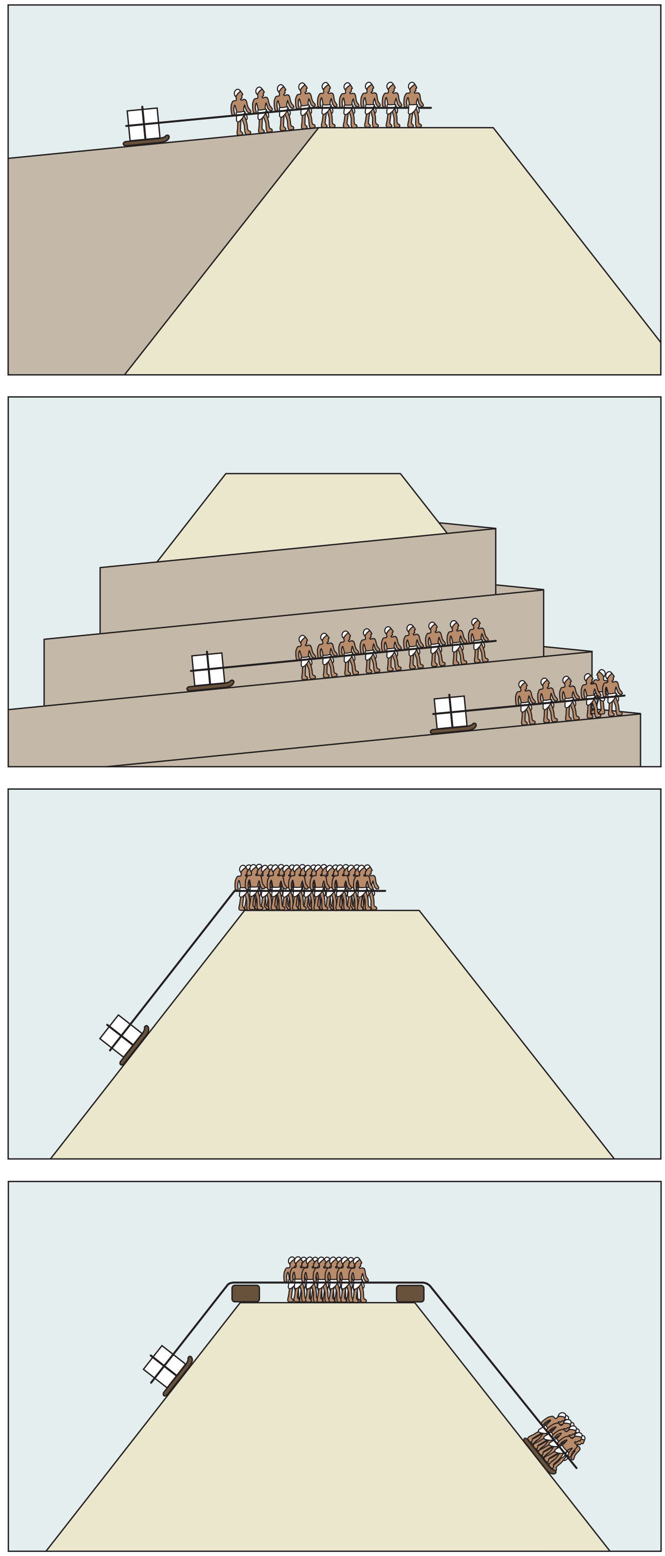

Figure 2. Earthen ramps that approach a pyramid directly or spiral up it (*top two panels*) have been proposed as means by which workers delivered blocks of stone to build the Egyptian pyramids. Alternatively, teams of about 50 workers could have dragged the stones up the side of the pyramid itself on water-lubricated wooden sledges *(third panel from top)*. This approach would have taken much less time (3 minutes, as opposed to at least 40) to raise each block. Further, if opposing sledges were used *(bottom panel)*, workers could have counterweighted the load, greatly reducing the effort required of the men atop the rising pyramid.

force of friction from a standstill, the system initially would be in equilibrium. To raise the block of stone, a relatively small team standing atop the plateau would have been sufficient to haul the rope up the incline. This would be so because the men in the sledge counterbalanced some of the weight of the stone, and because the weight of the two sledges counterbalanced each other, so that all the hauling team would have had to work against was part of the weight of the stone and that of the rope itself and the friction between the sledge and the inclined plane. Using Edwards's assumptions about the magnitude of these forces, a simple calculation shows that the hauling and lifting could have been done by an active team of fewer than a dozen men.

As the sledge loaded with the block of stone ascended (and the sledge loaded with men descended), the effort required of the hauling crew would have diminished, since some of the rope, and its weight, would have been transferred from one face of the pyramid to the other. In fact, the hauling crew would have to be alert at all times to the changing load, especially when there was more rope on the downslope than on the up, lest the sledges accelerate too much. After the experience of hundreds, if not thousands of stones raised in the construction of the pyramid, the teams could be expected to have gained a pretty good feel for the rope at its various stages of deployment.

A slight variation of the counterbalance scheme could also have been employed. In this case, with the empty sledge resting on the incline just over the edge of the plateau, workers could have climbed onto it one at a time until their accumulated weight was enough to get the sledge moving downward, thus pulling the stone-laden sledge up the other side of the pyramid. Once movement began, the weight of the worker-bearing sledge would overbalance that of the stone-bearing one because of the diminished friction force, so team members on the plateau would have to hold back the rope to keep the motion in check.

Regardless of the specific technique, one disadvantage of the double-sledge method would have been the fact that about a dozen workers would have ridden down with the counterweighting sledge. However, because only about two dozen members of a team would have been actively engaged in each lifting cycle, in the meantime other members of Edwards's team of 50 could have climbed back up to spell the workers on the top. (As Edwards indicates, it would have been wise if at least a portion of one of the faces of the pyramid not being used as inclined planes were left unfaced to provide a "stairway" by which workers could climb to and from the plateau.) In fact, with the counterbalancing scheme operating at steady state, after each haul—which would have occurred alternately on opposite faces—the rope crew could have served as ballast for the counterweight sledge, thus providing variety to their toil and perhaps not a little pleasure in getting to sit and ride down the side of the pyramid.

The counterbalanced-sledge system would have had another advantage over Edwards's, in that fewer workers would have to be atop the plateau at any given time. This would have been especially important as the pyramid rose, with the corresponding diminution of the plateau area. On the other hand, it might appear that at the early stages in the construction process, when the area of the plateau provided ample space for two large crews to work simultaneously on either side of a slipway, that fewer stones might be raised in a given period of time, since sledges and the connecting ropes would be occupying both ends of the slipway. However, during the time that the sledge atop was being moved across the plateau and unloaded, the empty one at the base could simultaneously have been being replaced by another from the quarry. Again following Edwards's estimates of the time to complete the various parts of the cycle, and allowing for the fact that subteams were available to carry out various tasks at the same time, the cycle time could easily have been halved by employing the double- or counterbalanced-sledge system. This would mean that Edwards's overall estimates for construction time need not be significantly affected.

There is clear evidence that the Egyptians understood the advantages of counterbalancing, and it appears in the same tomb painting that confirmed Edwards's calculations involving manpower and coefficient of friction. Depicted beneath the sledge carrying the statue of Djehutihotep are three men carrying what are assumed to be jars of liquid ready to replenish that being poured in front of the sledge by the man riding with the statue. The purpose of the liquid was, of course, to reduce the coefficient of friction between the wooden sledge and the surface over which it was being pulled. The reserve jugs are suspended from a shoulder yoke, an elementary example of counterbalancing. Given this ancient evidence of an appreciation of the advantages of counterbalancing, it is not unreasonable to assume that the principle might have been employed in the construction of the pyramids.

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